$$\langle s+1\rangle = \langle s\rangle + 1 = \frac{1}{e^{\alpha}-1} + 1 = \frac{1}{1-e^{-\alpha}}$$

$$\langle s\rangle = \frac{1}{e^{\alpha}-1} = \frac{e^{-\alpha}}{1-e^{-\alpha}}$$

$$\Rightarrow \quad \langle s+1\rangle \log\langle s+1\rangle = \frac{1}{1-e^{-\alpha}}(-)\log(1-e^{-\alpha})$$

$$\langle s\rangle \log\langle s\rangle = \frac{e^{-\alpha}}{1-e^{-\alpha}}\left(-\alpha - \log(1-e^{-\alpha})\right)$$

$$\Rightarrow \quad \langle s+1\rangle \log\langle s+1\rangle - \langle s\rangle \log\langle s\rangle$$

$$= \log(1-e^{-\alpha})\underbrace{\left(\frac{-1}{1-e^{-\alpha}} + \frac{e^{-\alpha}}{1-e^{-\alpha}}\right)}_{=-1}$$

$$+\frac{\alpha\, e^{-\alpha}}{1-e^{-\alpha}}.$$

$$= -\log(1-e^{-\alpha}) + \frac{\alpha}{e^{\alpha}-1} = \sigma \qquad \text{q.e.d.}$$

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