2) a) $\varepsilon_n = \frac{\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2 n^2.$

b) $F = -\tau \log Z.$

$$Z = \frac{1}{N!} Z_1^N.$$

$$Z_1 = \int_0^\infty dn\, e^{-\frac{\varepsilon_n}{\tau}} = \left(\frac{L}{\pi}\right)\sqrt{\frac{2m\tau}{\hbar^2}} \int_0^\infty dx \exp(-x^2)$$

$$= L\left(\frac{m\tau}{2\pi\hbar^2}\right)^{1/2} = \frac{n_Q}{n}.$$

$$Z_N = \frac{1}{N!} Z_1^N$$

$$\Rightarrow F = -\tau N\left[\log Z_1 - \log N + 1\right].$$

c) $S = -\frac{\partial F}{\partial \tau} = N\left[\log Z_1 - \log N + 1\right] + \frac{1}{2}\tau N \frac{1}{\tau}$

$$= N\left[\log Z_1 - \log N + \frac{3}{2}\right]$$

d). $\varepsilon = \frac{\hbar^2}{2m}\left(\frac{\pi^2}{L^2}\right)n^2.$

$$\Rightarrow n = \sqrt{\left(\frac{L^2}{\pi^2}\right)\frac{2m}{\hbar^2}}\ \sqrt{\varepsilon}.$$

$$\Rightarrow dn = \frac{L}{\pi}\sqrt{\frac{m}{2\hbar^2}}\ \frac{1}{\sqrt{\varepsilon}}\, d\varepsilon$$

$$\Rightarrow D(\varepsilon) = \frac{L}{\pi}\sqrt{\frac{m}{2\hbar^2}}\ \frac{1}{\sqrt{\varepsilon}}$$