4)

a) $\lambda = e^{\frac{\mu}{\tau}}$

$$\Rightarrow Z = \lambda^2 e^{\frac{2\delta}{\tau}} + \lambda e^{-\frac{1}{2}\frac{\Delta}{\tau}} + \lambda^3 e^{-\frac{\Delta}{2\tau}} + \lambda^2 e^{-\frac{2\delta}{\tau}}.$$

b) $$\langle N \rangle = \frac{2\lambda^2 e^{\frac{2\delta}{\tau}} + \lambda e^{-\frac{1}{2}\frac{\Delta}{\tau}} + 3\lambda^3 e^{-\frac{\Delta}{2\tau}} + 2\lambda^2 e^{-\frac{2\delta}{\tau}}}{Z}.$$

c). $\langle N \rangle = 1$:

$$2\lambda^2 e^{\frac{2\delta}{\tau}} + \cancel{\lambda e^{-\frac{1}{2}\frac{\Delta}{\tau}}} + 3\lambda^3 e^{-\frac{\Delta}{2\tau}} + 2\lambda^2 e^{-\frac{2\delta}{\tau}} \overset{?}{=}$$

$$\lambda^2 e^{\frac{2\delta}{\tau}} + \cancel{\lambda e^{-\frac{1}{2}\frac{\Delta}{\tau}}} + \lambda^3 e^{-\frac{\Delta}{2\tau}} + \lambda^2 e^{-\frac{2\delta}{\tau}}.$$

$$\Leftrightarrow \lambda^2 e^{\frac{2\delta}{\tau}} + 2\lambda^2 e^{-\frac{\Delta}{2\tau}} + \lambda e^{-\frac{2\delta}{\tau}} = 0.$$

$$\Leftrightarrow e^{\frac{2\delta}{\tau}} + e^{-\frac{2\delta}{\tau}} + 2\lambda^2 e^{-\frac{\Delta}{2\tau}} = 0.$$

$$2\cosh\left(\frac{2\delta}{\tau}\right) = -2\lambda^2 e^{-\frac{\Delta}{2\tau}}$$ does not work.