Solutions Midterm

Problem 1

a) $g(p) = A\, p \exp(-\alpha p)$

$$\Rightarrow P(p) = \frac{g(p)}{\int_0^\infty dp\, g(p)}$$ (5 points)

$$\int_0^\infty dp\, g(p) = A \int_0^\infty dp\, p \exp(-\alpha p) = A\left(-\frac{\partial}{\partial \alpha}\right) \int_0^\infty dp \exp(-\alpha p) =$$

$$= A\left(-\frac{\partial}{\partial \alpha}\right) \frac{1}{\alpha} = \frac{A}{\alpha^2}$$ (3 points)

$$\Rightarrow \boxed{P(p) = \alpha^2 p \exp(-\alpha p)}$$ (2 points)

b) $$\langle \varepsilon \rangle = \left\langle \frac{p^2}{2m} \right\rangle = \int_0^\infty \frac{p^2}{2M} P(p)$$ (5 points)

$$= \frac{1}{2M} \alpha^2 \int_0^\infty p^3 \exp(-\alpha p) = \frac{\alpha^2}{2M} \left(-\frac{\partial}{\partial \alpha}\right)^3 \frac{1}{\alpha}$$

$$= \frac{\alpha^2}{2M}\, 2 \times 3\, \frac{1}{\alpha^4} = \frac{3}{M} \frac{1}{\alpha^2}$$ (8 point)

$$\langle \varepsilon \rangle = \frac{3}{M} \frac{1}{\alpha^2}$$ (2 points)