Problem 3

a) $Z_1 = \sum_{n_x, n_y} e^{-\varepsilon_{n_x,n_y}/\tau}$ (5 points)

$$\simeq \int_0^\infty dn_x \int_0^\infty dn_y \; e^{-\alpha^2(n_x^2+n_y^2)}$$

$$\alpha^2 = \frac{\hbar^2}{2M}\frac{\pi^2}{L^2}\frac{1}{\tau} \quad ; \quad x \equiv \alpha n_x$$

$$= \left(\frac{1}{\alpha}\int_0^\infty dx\, e^{-x^2}\right)^2 = \frac{1}{\alpha^2}\frac{\pi}{4}.$$

$$\Rightarrow Z_1 = \frac{2ML^2\tau\pi}{4\hbar^2\pi^2} = \frac{ML^2\tau}{2\pi\hbar^2}$$ (8 points)

$\Rightarrow Z_1 = n_Q{}^n V^1$, 2 dimensions $\Rightarrow V = L^2$

$$\Rightarrow n_Q = \left(\frac{M\tau}{2\pi\hbar^2}\right)$$ (5 points).

b) $Z_N = \frac{1}{N!} Z_1^N$ (4 points)

double counting (2 points)
+ no interactions (2 points) (2 out of 3 is enough)
+ $n \ll n_Q$