Problem 2.1

$g(U) = C\, U^{3N/2}$, $C = \text{const.}$, $N = \#$ of particles.

a) Show $U = \frac{3}{2} N \tau$

$$\sigma = \ln g = \ln C + \frac{3N}{2} \ln U$$

$$\frac{1}{\tau} = \frac{\partial}{\partial U} \sigma = \frac{3N}{2} \frac{1}{U} \iff \boxed{U = \frac{3}{2} N \tau}$$

b) $$\frac{\partial^2 \sigma}{\partial^2 U} = \frac{3N}{2} - \frac{1}{U^2} < 0$$

Problem 2.2

Find fractional magnetization:

$$\frac{M}{Nm} = \frac{2\langle s \rangle}{N}$$

Use: $\sigma(s) = \log g(N, 0) - 2 s^2 / N$.

$$U = -2 \langle s \rangle m B \Rightarrow s = -\frac{U}{2mB}$$

$$\Rightarrow \sigma(U) = \sigma_0 - \frac{U^2}{2m^2 B^2 N}.$$

$$\Rightarrow \frac{1}{\tau} = \frac{\partial \sigma}{\partial U} = -\frac{U}{m^2 B^2 N} = \frac{2s}{mBN}$$

$$\Rightarrow \boxed{\frac{M}{Nm} = \frac{2\langle s \rangle}{N} = \frac{1}{\tau} mB}$$