Problem 2.3

$$g(N, u) = \frac{(N+u-1)!}{u!\,(N-1)!}$$

$$\sigma(N,u) = \log\left(g(N,u)\right)$$

$$= \log(N+u-1)! - \log u! - \log(N-1)!$$

Use $\log M! = M \log M - M$.

$\Rightarrow$

$$\sigma = (N+u)\log(N+u) - \cancel{(N+u)} - u\log u + \cancel{u} - N\log N + \cancel{N}$$

$$= N\log\left(\frac{N+u}{N}\right) + u\log\left(\frac{N+u}{u}\right)$$

$$= \underline{N\log\left(1+\frac{u}{N}\right) + u\log\left(1+\frac{N}{u}\right)}$$

b) $U = u\hbar\omega \Rightarrow u = U/\hbar\omega$

$$\sigma = N\log\left(1+\frac{U}{\hbar\omega N}\right) + \frac{U}{\hbar\omega}\log\left(1+\frac{N\hbar\omega}{U}\right)$$

$\Rightarrow$

$$\frac{1}{T} = \frac{\partial\sigma}{\partial U} = N\frac{1}{1+\frac{U}{\hbar\omega N}}\frac{1}{\hbar\omega N} + \frac{1}{\hbar\omega}\log\left(1+\frac{N\hbar\omega}{U}\right)$$

$$+ \frac{U}{\hbar\omega}\frac{1}{1+\frac{N\hbar\omega}{U}}\left(-\frac{N\hbar\omega}{U^2}\right)$$