(b) As I showed, $\omega_n \propto \frac{1}{V^{1/3}}$, so $\omega_n = \alpha_n V^{-1/3}$, $\alpha_n$ – some quant independent of $V$.

Then $\left(\frac{\partial \omega_n}{\partial V}\right)_\sigma = \alpha_n \left(\frac{\partial V^{-1/3}}{\partial V}\right)_\sigma = -\frac{1}{3}\alpha_n V^{-4/3} = -\frac{1}{3}\alpha_n V^{-1/3} V^{-1} =$

$$= -\frac{1}{3}\frac{\omega_n}{V}$$

(c) Then $p = -\sum_j s_j \hbar \left(-\frac{1}{3}\frac{\omega_j}{V}\right) = \frac{1}{3V}\sum_j s_j \hbar \omega_j = \frac{U}{3V}$

(d) From Stefan – Boltzmann law,

$$p = \frac{1}{3}\left(\frac{U}{V}\right) = \frac{1}{3}\frac{\pi^2}{15\hbar^3 c^3}\tau^4 = 6.95\times 10^{75}\tau^4$$

For an ideal gas, $pV = NkT = N\tau$, so $p = \frac{N}{V}\tau = n\tau =$

$$= 6.02\times 10^{23}\frac{1}{mol}\ \frac{1\,mol}{(0.08\,m)^3}\tau = 6.02\times 10^{29}\tau$$

For the pressures to be equal,

$$6.95\times 10^{75}\tau^4 = 6.02\times 10^{29}\tau$$

$$\tau = \left(\frac{6.02\times 10^{29}}{6.95\times 10^{75}}\right)^{1/3} = 4.42\times 10^{-16}\,J = kT$$

$$T = \frac{4.42\times 10^{-16}\,J}{1.38\times 10^{-23}\,J/K} = 3\times 10^7\,K$$