Solutions Chapter 4

1). $$\langle s \rangle = \frac{1}{e^{\hbar\omega/\tau} - 1}$$

polarization

$$= U = \sum_n \langle s_n \rangle = \frac{2}{8} \int 4\pi n^2 \frac{1}{e^{\hbar\omega_n/\tau} - 1} \quad ; \quad \omega_n = \frac{n\pi c}{L}$$

only positive octant of space counts.

$$= \pi \frac{L^3 \tau^3}{(\hbar\pi c)^3} \int_0^\infty \frac{x^2\,dx}{e^x - 1}$$

$$= \pi \frac{L^3 \tau^3}{(\hbar\pi c)^3} \int_0^\infty x^2\,dx\; e^{-x} \left[1 + e^{-x} + e^{-2x} + \dots \right]$$

$$= \pi \frac{L^3 \tau^3}{(\hbar\pi c)^3}\, 2 \left[1 + \frac{1}{8} + \frac{1}{27} + \frac{1}{64} + \frac{1}{125} + \frac{1}{216} + \dots \right]$$

$$\boxed{U = \frac{2.4}{\pi^2}\, V \left(\frac{\tau}{\hbar c}\right)^3}$$