a) $\mathcal{Z} = 1 + \lambda e^{0} + \lambda e^{-\varepsilon/\tau} =$

$$1 + \lambda + \lambda e^{-\varepsilon/\tau}$$

b) $$\langle N \rangle = \tau \frac{\partial \log \mathcal{Z}}{\partial \mu} = \lambda \frac{\partial \log \mathcal{Z}}{\partial \lambda} = \frac{\lambda + \lambda e^{-\varepsilon/\tau}}{\mathcal{Z}}$$

c) $$\langle N(\varepsilon) \rangle = \frac{\lambda e^{-\varepsilon/\tau}}{\mathcal{Z}}$$ from b)

d) $$\langle \varepsilon N(\varepsilon) \rangle = \sum_{n \geq 0} \varepsilon \lambda e^{-\varepsilon/\tau} = \varepsilon \lambda e^{-\varepsilon/\tau}$$

only 1 state contributes.

e) $$\mathcal{Z} = \sum_{n \geq 0} \exp\left[-(\varepsilon - \mu N)/\tau\right]$$

$$= e^{-(0-0)/\tau} + e^{-(0-\mu)/\tau} + e^{-(\varepsilon-\mu)/\tau}$$

$$+ e^{-(0-\mu)/\tau} e^{-(\varepsilon-\mu)/\tau}$$

$$= 1 + \lambda + \lambda \exp(-\varepsilon/\tau) + \lambda^2 \exp(-\varepsilon/\tau)$$

$$= \underline{(1+\lambda)(1 + \lambda \exp(-\varepsilon/\tau))}$$