) Show that $\mu = \left(\frac{\partial U}{\partial N}\right)_{T,V}$.

Assume $dV = 0$

$$dU = \left(\frac{\partial U}{\partial T}\right)_N dT + \left(\frac{\partial U}{\partial N}\right)_T dN$$

Now isothermal change of energy.

$$(\delta U)_T = \underbrace{\left(\frac{\partial U}{\partial T}\right)_N}_{\tau} (\delta T)_T + \left(\frac{\partial U}{\partial N}\right)_T \delta N_T$$

$$\Rightarrow \left(\frac{\delta U}{\delta N}\right)_T = \tau \left(\frac{\delta T}{\delta N}\right)_T + \left(\frac{\partial U}{\partial N}\right)_T \qquad (*)$$

Now we know:

$$\mu = \left(\frac{\partial F}{\partial N}\right)_T = \left(\frac{\partial U}{\partial N}\right)_T - \tau\left(\frac{\partial S}{\partial N}\right)_T$$

$$\Leftrightarrow \left(\frac{\partial U}{\partial N}\right)_T = \tau\left(\frac{\partial S}{\partial N}\right) + \mu$$

inserting in $(*)$

$$\cancel{\tau\left(\frac{\partial S}{\partial N}\right)} + \mu = \cancel{\tau\left(\frac{\partial S}{\partial N}\right)_T} + \left(\frac{\partial U}{\partial N}\right)_T$$

$$\boxed{\left(\frac{\partial U}{\partial N}\right)_T = \mu}$$