Notice that $n_Q = \left(\frac{M\tau}{2\pi\hbar^2}\right)^{3/2} = n_{QA} = n_{QB}$, so

$$(1) \Rightarrow \sigma_i = N\left(\ln\left(\frac{n_Q}{n}\right)^2 + \frac{10}{2}\right) = N\left(2\ln\frac{n_Q}{n} + 2\cdot\frac{5}{2}\right) = \sigma_f \text{ , QED}$$

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The result, called the Gibbs paradox, is not that strange. We always expect the entropy to increase during a non-reversible process and remain the same during a reversible one.

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(a) We know that $p_s = \frac{e^{-\varepsilon_s/\tau}}{Z}$ and

(★) $$U \equiv \langle \varepsilon \rangle = \sum_s \varepsilon_s p_s$$

Also, (3.26) $\Rightarrow$ $p = -\left(\frac{\partial U}{\partial V}\right)_{\sigma,N}$

Choose a slow expansion as your process. As discussed before, from quantum mechanics we know that if the system is isolated during this expansion it will remain in the same quantum state and so the multiplicity (= degeneracy) will remain the same, which means $\sigma = \text{const}$. What is important for us now is that the multiplicities of the neighbouring states also remains the same (it remains the same for all states). That means $d\sigma = \sigma(\varepsilon_s + d\varepsilon_s) - \sigma(\varepsilon_s)$ also doesn't change (even though ε_s themselves do change!). But

$$\frac{1}{\tau} = \frac{d\sigma}{d\varepsilon_s} \Rightarrow d\sigma = \frac{d\varepsilon_s}{\tau} = \text{const}$$

Consider two neighbouring states with energies ε_s and $\varepsilon_s + d\varepsilon_s$. The ratio of their probabilities is:

$$\frac{p_{s+1}}{p_s} = \frac{e^{-(\varepsilon_s + d\varepsilon_s)/\tau}}{e^{-\varepsilon_s/\tau}} = e^{-d\varepsilon_s/\tau} = \text{const.}$$

But if the relative probabilities of neighbouring states doesn't change,