(5) luate $Z_{int} \overset{(6.44)}{=} \sum_{int} e^{-\varepsilon_{int}/\tau}$. We only have two internal states. Assuming that the energy of the lower one is zero (only energy differences matter physically, although choosing a different zero of energy would lead to shifting μ and F by a constant), we get:

$$Z_{int} = 1 + e^{-\Delta/\tau}$$

(a) $\mu = \tau\left(\ln\frac{n}{n_Q} - \ln Z_{int}\right)$ (2)

$(F_{int} = -N\tau \ln Z_{int})$

(b) $F = F_{usual} + F_{int} = N\tau\left(\ln\frac{n}{n_Q} - 1 - \ln Z_{int}\right)$ (3)

(c) $\sigma = \sigma_{usual} + \sigma_{int} = N\left(\ln\frac{n_Q}{n} + \frac{5}{2}\right) - \left(\frac{\partial F_{int}}{\partial \tau}\right)_V$ (2)

$$\left(\frac{\partial F_{int}}{\partial \tau}\right)_V = \frac{\partial}{\partial \tau}\left(-N\tau \ln\left(1+e^{-\Delta/\tau}\right)\right) =$$

$$= -N\left[\ln\left(1+e^{-\Delta/\tau}\right) + \frac{\tau}{1+e^{-\Delta/\tau}}\, e^{-\Delta/\tau}\frac{\Delta}{\tau^2}\right]$$

$$= -N\left[\ln Z_{int} + \frac{\Delta/\tau\, e^{-\Delta/\tau}}{Z_{int}}\right]$$

$$\sigma = N\left[\ln\frac{n_Q}{n} + \frac{5}{2} - \ln Z_{int} - \frac{\Delta/\tau\, e^{-\Delta/\tau}}{Z_{int}}\right]$$

(d) $p \underset{\text{summary, Ch. 6}}{=} -\left(\frac{\partial F}{\partial V}\right)_{\tau,N} = p_{usual} - \underbrace{\cancel{\left(\frac{\partial F_{int}}{\partial V}\right)_{\tau,N}}}_{0} = p_{usual} = n\tau$ (2)

$= \frac{N}{V}\tau$

$n_Q = \left(\frac{M\tau}{2\pi\hbar^2}\right)^{3/2}$

(6) (e) By definition, $C_p = \left(\frac{\delta Q}{d\tau}\right)_p = \tau\left(\frac{\partial \sigma}{\partial \tau}\right)_p$

We need $\left(\frac{\partial Z_{int}}{\partial \tau}\right)_p = \frac{dZ_{int}}{d\tau} = e^{-\Delta/\tau}\frac{\Delta}{\tau^2}$; also $\frac{n_Q}{n} = \frac{n_Q\tau}{n\tau} = \frac{n_Q}{p}\tau = \text{const}\,\tau^{5/2}$

So $C_p = \tau\left(\frac{\partial \sigma}{\partial \tau}\right)_p = N\tau\left[\frac{\partial\left(\ln \text{const} + \frac{5}{2}\ln\tau\right)}{\partial\tau} - \frac{1}{Z_{int}}\frac{\partial Z_{int}}{\partial \tau} - \frac{\left(\frac{-\Delta}{\tau^2}e^{-\Delta/\tau} + \frac{\Delta}{\tau}e^{-\Delta/\tau}\frac{\Delta}{\tau^2}\right)Z_{int}}{Z_{int}^2}\right.$

$-\frac{\Delta}{\tau}e^{-\Delta/\tau}\frac{\partial Z_{int}}{\partial \tau}$

Consider the last term in the brackets. It's equal to: