$$-\frac{1}{Z_{int}^2}\left(\left(\frac{-\Delta}{\tau^2}e^{-\Delta/\tau}+\frac{\Delta^2}{\tau^3}e^{-\Delta/\tau}\right)\left(1+e^{-\Delta/\tau}\right)-\frac{\Delta}{\tau}e^{-\Delta/\tau}e^{-\Delta/\tau}\frac{\Delta}{\tau^2}\right)=$$

$$=-\frac{1}{Z_{int}^2}\left(e^{-\Delta/\tau}\left(\frac{\Delta^2}{\tau^3}-\frac{\Delta}{\tau^2}\right)+e^{-2\Delta/\tau}\left(\cancel{\frac{\Delta^2}{\tau^3}}-\frac{\Delta}{\tau^2}-\cancel{\frac{\Delta^2}{\tau^3}}\right)\right)$$

Then the last two terms together are:

$$-\frac{Z_{int}\frac{\partial Z_{int}}{\partial\tau}}{Z_{int}^2}-\frac{1}{Z_{int}^2}(\cdots)=-\frac{1}{Z_{int}^2}\left(e^{-\Delta/\tau}\left(\cancel{\frac{\Delta}{\tau^2}}+\frac{\Delta^2}{\tau^3}-\cancel{\frac{\Delta}{\tau^2}}\right)+\right.$$

$$\left.+e^{-2\Delta/\tau}\left(\cancel{\frac{\Delta}{\tau^2}}-\cancel{\frac{\Delta}{\tau^2}}\right)\right)=-\frac{\Delta^2/\tau^3\, e^{-\Delta/\tau}}{Z_{int}^2}$$

So finally,

$$C_p=N\tau\left[\frac{5}{2}\frac{1}{\tau}-\frac{\Delta^2/\tau^3\, e^{-\Delta/\tau}}{Z_{int}^2}\right]=$$

$$=\frac{5}{2}N-\left(\frac{\Delta}{\tau Z_{int}}\right)^2 e^{-\Delta/\tau}$$

As usual, there are lots of ways to write down the answer. Looking back, I see that it might have been easier to define the zero of energy midway between the energy levels so that

$$Z_{int}=e^{\Delta/2\tau}+e^{-\Delta/2\tau}=2\cosh\left(\frac{\Delta}{2\tau}\right)$$

Then, for example,

$$\left(\frac{\partial F_{int}}{\partial\tau}\right)_V=\frac{\partial}{\partial\tau}\left(-N\tau\ln\left(2\cosh\left(\frac{\Delta}{2\tau}\right)\right)\right)=-N\left(\ln\left(2\cosh\left(\frac{\Delta}{2\tau}\right)\right)+\frac{2\tau\sinh\left(\frac{\Delta}{2\tau}\right)}{2\cosh\left(\frac{\Delta}{2\tau}\right)}\left(-\frac{\Delta}{2\tau^2}\right)\right)$$

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(15) (4.) Kittel 6.13

(a) $$\mathcal{Z}=\sum_{\text{all } s}\lambda^N e^{-\varepsilon_s/\tau}=\sum_N\lambda^N\sum_{\substack{s \text{ with}\\ \text{given } N}}e^{-\varepsilon_s/\tau}=\sum_N\lambda^N Z_N=$$

$$=\sum_N\frac{1}{N!}(\lambda n_Q V)^N=e^{\lambda n_Q V}\quad,\text{ since } e^x=\sum_n\frac{1}{n!}x^n$$ QED

(b) $$P(N)=\sum_{\substack{s \text{ with}\\ \text{given } N}}P_s=\sum_{\substack{s \text{ with}\\ \text{given } N}}\frac{\lambda^N e^{-\varepsilon_s/\tau}}{\mathcal{Z}}=\frac{\lambda^N}{\mathcal{Z}}Z_N=\frac{1}{N!}\frac{(\lambda n_Q V)^N}{\mathcal{Z}}=$$

P