Solutions Test final

1) harmonic oscillator $\varepsilon = \hbar\omega n$

$$\sum_n \longrightarrow \int dn = \int D(\varepsilon)\, d\varepsilon .$$

$$\varepsilon = \hbar\omega n \Rightarrow d\varepsilon = \hbar\omega\, dn$$

$$\Rightarrow D(\varepsilon) = \frac{1}{\hbar\omega}$$

2) $$Z_1 = \frac{V}{h^3} \int_{-\infty}^{\infty} dp_x \int_{-\infty}^{\infty} dp_y \int_{-\infty}^{\infty} dp_z \exp\left(-\frac{\varepsilon}{\tau}\right)$$

$$= \frac{V}{h^3}\, 4\pi \int p^2 dp \exp\left(-\frac{cp}{\tau}\right)$$

spherical coordinates.

$$= \frac{V}{h^3}\, 4\pi \frac{\tau^3}{c^3} \underbrace{\int_0^{\infty} x^2 dx \exp(-x)}_{2} \quad ; \quad x = \frac{cp}{\tau}$$

$$= 2\frac{V}{h^3}\, 4\pi \frac{\tau^3}{c^3}$$

$$= 8\pi \frac{V}{h^3} \frac{\tau^3}{c^3}$$

b) $$F = -\tau \log Z_N = -\tau \log \frac{Z_1^N}{N!} = -\tau\left[N \log Z_1 - N \log N + N\right]$$

$$\sigma = -\frac{\partial F}{\partial \tau} = +\left[N \log Z_1 - N \log N + N\right] + \tau \frac{N}{Z_1} \frac{\partial Z_1}{\partial \tau}$$

$$U = F + \tau\sigma = \tau \frac{N}{Z_1} \frac{\partial Z_1}{\partial \tau} = 3 N \tau$$

c) $$\sigma = -\tau N \left[\log\left(8\pi \frac{h \tau^3}{(hc)^3}\right) + 4\right]$$